International Journal of Theoretical Physics, Vol. 39, No. 7, 2000

Eternal Inflation, Black Holes, and the Future of Civilizations

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Received May 17, 2000

We discuss the large-scale structure of the universe in inflationary cosmology and the implications that it may have for the long-term future of civilizations. Although each civilization is doomed to perish, it may be possible to transmit its accumulated knowledge to future civilizations. We consider several scenarios of this sort. If the cosmological constant is positive, it eventually dominates the universe and bubbles of inflationary phase begin to nucleate at a constant rate. Thermalized regions inside these inflating bubbles will give rise to new galaxies and civilizations. It is possible in principle to send a message to one of them. It might even be possible to send a device whose purpose is to recreate an approximation of the original civilization in the new region. However, the message or device will almost certainly be intercepted by black holes, which nucleate at a much higher rate than inflating bubbles. Formation of new inflating regions can also be triggered by gravitational collapse, but again the probability is low, and the number of attempts required for a positive outcome is enormous. The probability can be higher if the energy scale of inflation is closer to the Planck scale, but a high energy scale produces a tight bound on the amount of information that can be transmitted. One can try to avoid quantum tunneling altogether, but this requires a violation of quantum inequalities which constrain the magnitude of negative energy densities. However, the limits of validity of quantum inequalities are not clear, and future research may show that the required violation is in fact possible. Therein lies the hope for the future of civilizations.

1. ETERNAL INFLATION

Inflation is a period of accelerated expansion in the early universe. It is the only cosmological scenario that we have that can explain the large-

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0020-7748/00/0700-1887$18.00/0

scale homogeneity and flatness of the universe. During inflation the universe is expanded by a huge factor, so that we can see only a part of it, which is nearly homogeneous and flat. The inflationary expansion is driven by the potential $V(\phi)$ of a scalar field ϕ, which is called the inflaton. In Fig. 1 the inflaton is represented by a little ball that rolls down the potential hill. Near the top of the potential the slope is very small, so the roll is slow and $V(\varphi) \approx$ const. In this regime the universe expands exponentially,

$$a(t) \propto e^{Ht} \tag{1}$$

with the expansion rate determined by the height of the potential,

$$H^2 = \frac{8\pi}{3} V(\varphi) \tag{2}$$

(Throughout the paper we use natural units in which $\hbar = c = G = 1$.) When φ gets to the steep part of the potential, it starts oscillating about the minimum, and its energy gets dumped into relativistic particles. The particles quickly thermalize at a high temperature, and the following evolution is along the lines of the standard hot cosmological model. Thus, thermalization plays the role of the big bang in the inflationary scenario, and inflation prepares the initial conditions for the big bang (for a review of inflation, see ref. 1).

We hope that the shape of the potential $V(\varphi)$ will be determined from the theory of elementary particles, but our present understanding of particle physics is not sufficient for this task. Fortunately, some general features of inflation can be studied without a detailed knowledge of $V(\varphi)$.

One of the remarkable features of inflation is its eternal character. This is due to quantum fluctuations of the inflaton. On the flat portion of the potential, the force that drives φ down the hill is small and quantum fluctuations are important. The physics of the fluctuations is determined by the expansion rate H: the field φ experiences quantum jumps of magnitude $\delta\varphi \sim$

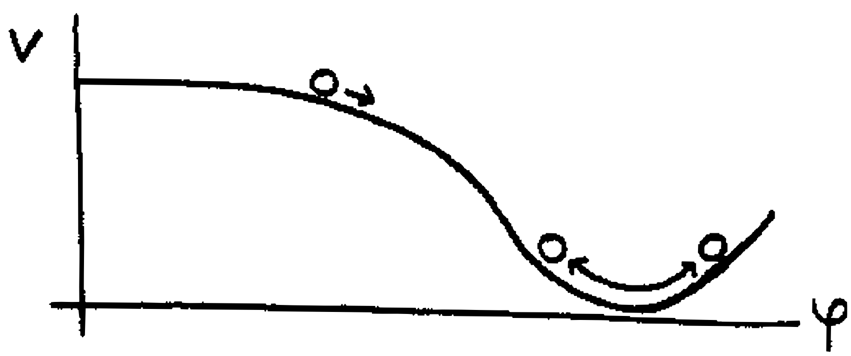

Fig. 1. The inflation potential.

$\pm H/2\pi$ on a time scale $\delta t \sim H^{-1}$. These jumps are not homogeneous in space: quantum fluctuations of φ are not correlated over distances larger than H^{-1}. In other words, fluctuations in nonoverlapping regions of size $\sim H^{-1}$ are independent of one another.

Since quantum fluctuations of φ are different at different locations, the inflaton does not get to the themalization point at the bottom of the hill simultaneously everywhere in space. In some rare regions, the fluctuations will keep φ at high values of the potential for much longer than it would otherwise stay there. Such regions will be "rewarded" by a large amount of expansion at a high rate H. The dynamics of the total volume of inflating regions in the universe is thus determined by two competing processes: the loss of inflating volume due to thermalization, and generation of new volume at the high rate sustained by quantum fluctuations. Analysis shows that the second process "wins" for a generic inflaton potential $V(\varphi)$, and the total inflating volume grows with time [2–4]. Thus, inflation never ends completely: at any time there are parts of the universe that are still inflating. The spatial distribution of inflating and thermalized regions in an eternally inflating universe is illustrated in Fig. 2. It was obtained in a numerical simulation

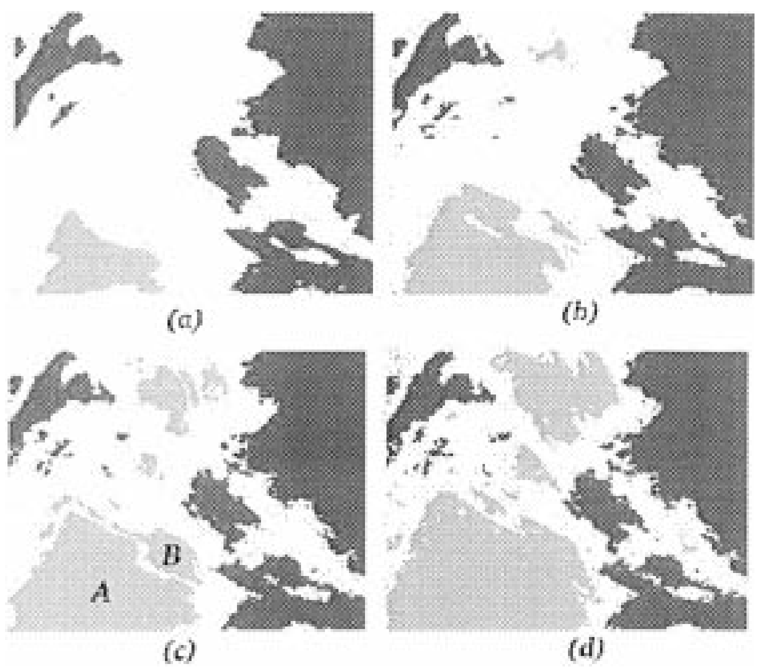

Fig. 2. A simulation for the double-well model at four consecutive moments of time. Inflating regions are white, while thermalized regions with $\phi = +\eta$ and $\phi = -\eta$ are shown with different shades of grey [5].

[5] for a double-well potential $V(\varphi) = \lambda(\varphi^2 - \eta^2)^2$. Inflating regions are white, and the two types of thermalized regions corresponding to the two minima at $\varphi = \pm\eta$ are shown with different shades of grey. Different types of thermalized regions will generally have different physical properties.

The spacetime structure of the universe in this model is illustrated in Fig. 3 using the same shading code. Now, the vertical axis is time and the horizontal axis is one of the spatial directions. The boundaries between inflating and thermalized regions play the role of the big bang for the corresponding thermalized regions. In the figure, these boundaries become nearly vertical at late times, so that it appears that they correspond to a certain position in space rather than to a certain moment of time. The reason is that the horizontal axis in Fig. 3 is the comoving distance, with the expansion of the universe factored out. The physical distance is obtained by multiplying by the expansion factor $a(t)$, which grows exponentially as we go up along the time axis. If we used the physical distance in the figure, the thermalization boundaries would "open up" and become nearly horizontal (but then it would be difficult to fit more than one thermalized region in the figure).

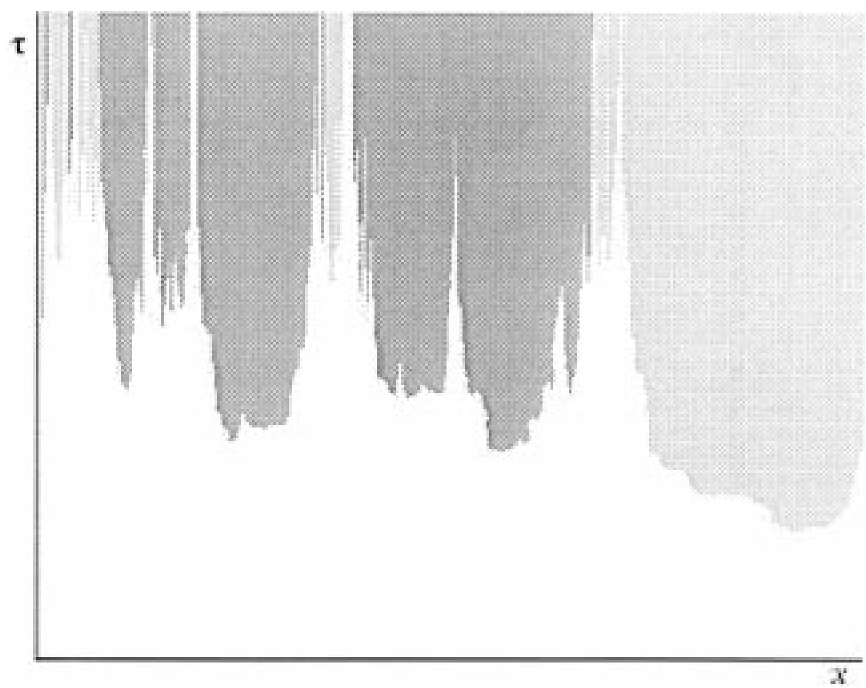

Fig. 3. Spacetime structure simulation for the double-well model. Inflating regions are white, while thermalized regions with $\phi = +\eta$ and $\phi = -\eta$ are shown with different shades of grey [5].

The spacetime structure of a thermalized region near the thermalization boundary is illustrated in Fig. 4. The thermalization is followed by a hot radiation era and then by a matter-dominated era during which luminous galaxies are formed and civilizations flourish. All stars eventually die, and thermalized regions become dark, cold, and probably not suitable for life (see the next section). Hence, civilizations are to be found within a layer of finite (temporal) width along the thermalization boundaries in Fig. 3.

For an observer inside one of the thermalized regions, the thermalization boundary (the "big bang") is in his past, so he cannot reach this boundary, no matter how fast he moves.[4] Even at the speed of light, it is impossible to send a signal that would cross the boundary and get into the inflating region. It follows that different thermalized regions are causally disconnected from one another: it is impossible to send signals between different regions, and the course of events in one region can be in no way affected by what is happening in another.

This conclusion may be avoided in the presence of a peculiar form of matter described by a field equation with a special nonlinear gradient term in which sound propagates faster than the speed of light [6]. The existence of such matter is consistent with the principles of the theory of relativity: the Lagrangian is perfectly Lorentz invariant, but the cosmological solutions select a preferred frame in which the speed of sound can be superluminal. In what follows, however, we shall disregard this somewhat exotic possibility. Note also that in order to get from one thermalized region to another, the

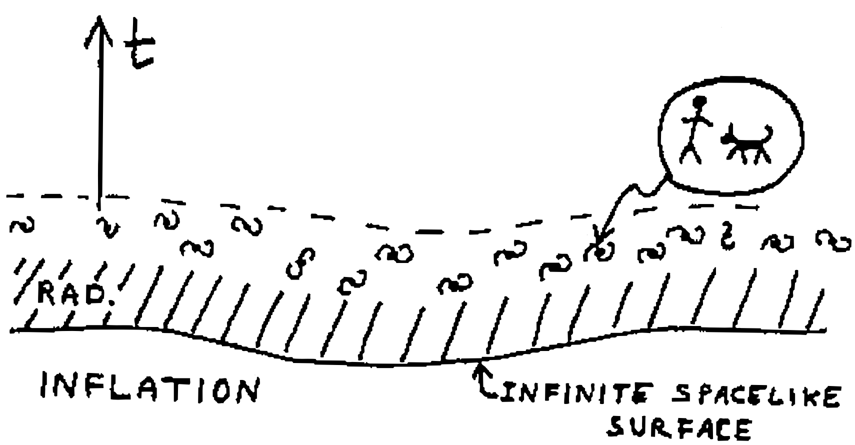

Fig. 4. Spacetime structure of a thermalized region near the thermalization boundary.

[4] Thermalization boundaries are infinite spacelike hypersurfaces. With an appropriate choice of the time coordinate, each thermalized region is an infinite subuniverse containing an infinite number of galaxies.

sound waves have to cross the inflating region that separates them. As a result, unless the speed of sound is so large that the trip can be made almost instantaneously, the wavelength may get stretched by an enormous expansion factor. In this case the period of the waves would become too long for even one oscillation during the lifetime of a star in the target region, and it is hard to see how such waves could transmit any information.

2. MESSAGES TO THE FUTURE

Let us now consider the future prospects for a civilization on a cosmological time scale. It appears very unlikely that a civilization can survive forever. Even if it avoids natural catastrophes and self-destruction, it will in the end run out of energy. The stars will eventually die, and other sources of energy (such as tidal forces) will also come to an end. Dyson [7] has argued that civilizations may still survive indefinitely into this cold and dark future of the universe by constantly reducing the rate of their energy consumption. This must be accompanied by a corresponding reduction in the rate of information processing. In the asymptotic future both rates become infinitesimal, but Dyson argued that the total amount of information processed by a civilization may still be infinite. The "subjective" lifetime of such a civilization would then also be infinite, as well as its physical lifetime. Nevertheless, a more detailed analysis of the problem has recently been given by Krauss and Starkman [8], with the conclusion that the steady decrease in the rate of information processing proposed by Dyson seems physically impossible to achieve. Thus, it appears that an eternal civilization is impossible, even in principle.

If we are doomed to perish, then perhaps we could send messages, or even representatives, to future civilizations? Those civilizations could also send messages to the future, and so on. We would then become a branch in an infinite "tree" of civilizations, and our accumulated wisdom would not be completely lost. Here we shall consider the feasibility of some scenarios of this sort.[5]

2.1. Recycling Universe

In an eternally inflating universe, new thermalized regions will continue to be formed in the arbitrarily distant future. These regions win go through radiation-and matter-dominated periods, will form galaxies and evolve civili-

[5] Linde [18] suggested that the message could be sent in the form of the values of the fundamental constants of nature. However, it is not clear that the constants in a newly created inflating region can be different from those in the parent universe, and if so, whether or not we can have any control over their values. We shall not consider this possibility in the present paper.

zations. However, as we discussed at the end of the preceding section, different thermalized regions are causally disconnected from one another and communication between them is impossible. We can of course send messages to other civilizations within our own thermalized region. But the resulting tree of civilizations would necessarily be finite, and its time span would be bounded by the same energetic factors that limit the lifetime of a single civilization.

The spacetime structure of the universe illustrated in Fig. 3 assumes that the vacuum energy density, otherwise known as the cosmological constant Λ, is equal to zero. However, observations of distant supernovae performed by two independent groups during the last year suggest that the universe is now expanding with an acceleration, indicating that Λ has a small, positive value [9]. A nonzero Λ, no matter how small, changes the large-scale structure of the universe [10]. As the universe expands, the density of matter goes down, $\rho_m \propto a^{-3}$, while the vacuum energy density remains constant, so eventually the universe gets dominated by the cosmological constant. After that it expands exponentially, as in Eq. (1), but at a very small rate $H_\Lambda = (\Lambda/3)^{1/2}$.

In a universe with $\Lambda > 0$, there is a finite constant probability for the inflaton field to tunnel quantum mechanically from its vacuum value, where $V(\varphi) = \Lambda/8\pi$, to the values in the inflationary range near the top of the potential. The tunneling occurs within a spherical volume of radius $\sim H_\Lambda^{-1}$ (a "bubble"), with a probability per unit volume per unit time $\mathcal{P} \propto \exp(S_b - 3\pi\Lambda^{-1})$ [11]. Here S_b is the action of the instanton solution responsible for the tunneling. (Note that $\mathcal{P}$ vanishes for $\Lambda = 0$.) Each inflating bubble develops into a fully fledged, eternally inflating region of the universe. In the course of its evolution, it forms an infinite number of thermalized regions, each containing an infinite number of galaxies. These thermalized regions later get dominated by the cosmological constant, with subsequent nucleation of new inflationary bubbles, and so on [10]. The large-scale structure of such a "recycling" universe is illustrated in Fig. 5.

The recycling nature of the universe opens the possibility of sending a message to a future civilization. All one needs is a very strong and durable container. One simply puts the message in the container and sends it into space. In due course, the universe gets dominated by the cosmological constant, and inflating bubbles begin to nucleate. The hope in that the container will be engulfed by one of the bubbles.

The problem with this scenario is that inflating bubbles are not the only things that can nucleate in a Λ-dominated universe. There is also a constant rate of nucleation of black holes, $\mathcal{P}_{\rm bh} \propto \exp(-\pi/\Lambda)$. For the value of $\Lambda \sim 10^{-122}$ suggested by observations, this is greater than the rate of bubble nucleation by a factor $\sim\exp(10^{122})$. Thus, the message-carrying container

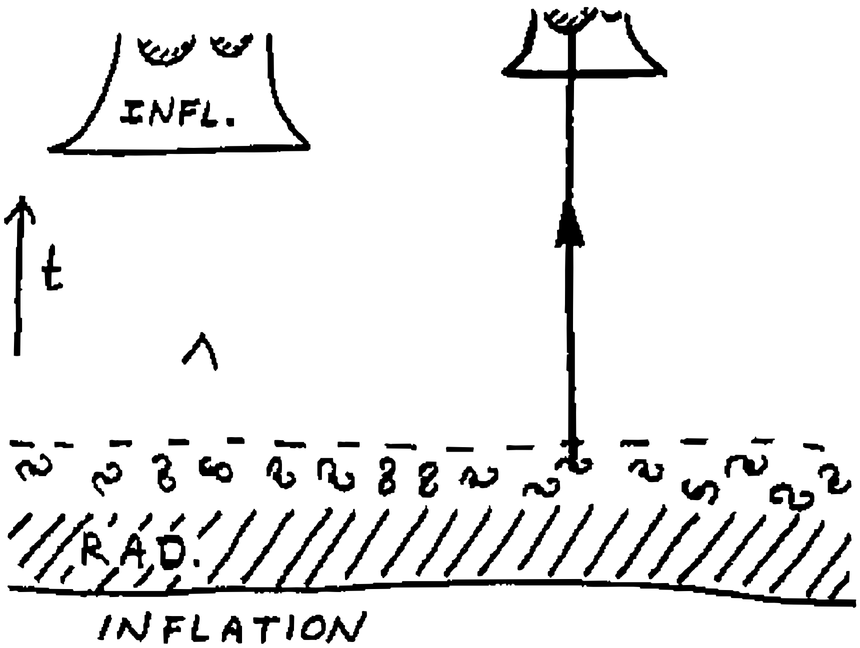

Fig. 5. Global structure of a recycling universe. After the universe has thermalized, the cosmological constant Λ drives an accelerated expansion with constant Hubble rate. This triggers the nucleation of "bubbles" of the new phase at a constant rate. Two of these bubbles are represented in the top part of the figure. In a "recycling" universe, a message whose worldline is indicated by a vertical arrow can travel from one civilization in the thermalized region at the bottom of the figure to another civilization which resides in a future thermalized region. Most messages, however, are intercepted by nucleating black holes.

will almost certainly be swallowed by a black hole. In order to beat the odds, one would have to send more than $\sim\exp(10^{122})$ containers.

2.2. Black Holes and an Upper Bound on Information

Instead of relying on bubble nucleation, one can take a more active approach.[6] Quantum nucleation of an inflating region can be triggered by a gravitational collapse in our part of the universe [12] (see Fig. 6). One has to generate an implosion of a small high-energy vacuum region, leading to gravitational collapse, with a message-carrying container at the implosion center. All one sees is the formation of a black hole; a new inflating region may or may not be inside it. The tunneling probability for the formation of such a region is [12]

[6] This possibility was suggested to us by E. Guendelman.

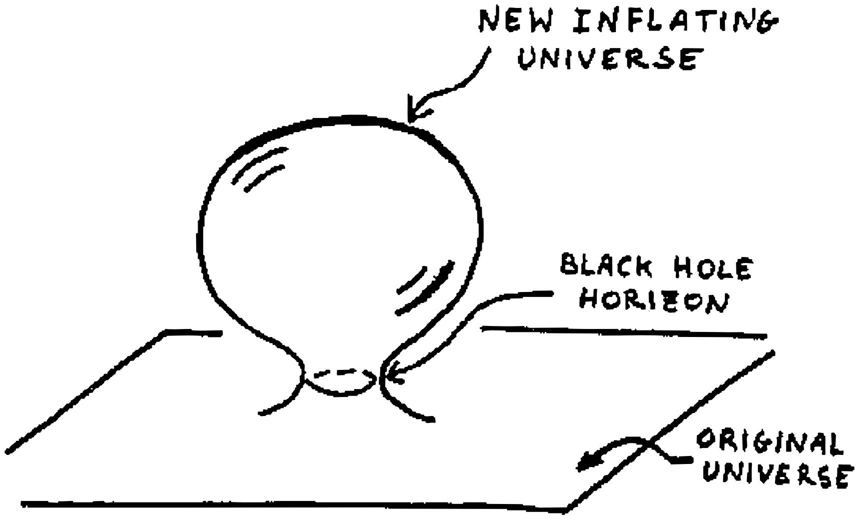

Fig. 6. A new inflating region can be triggered by gravitational collapse in our part of the universe. The figure represents a spacelike section of the resulting space-time. The inflating region eventually "pinches off" the original universe.

$$\mathcal{P} \propto \exp(-C/H^2) \tag{3}$$

where H is the expansion rate in the new inflating region and $C \sim 1$ is a constant coefficient. For a grand-unification-scale inflation, $H \sim 10^{-7}$ and $\mathcal{P} \sim \exp(-10^{14})$.

The exponential suppression of the tunneling disappears if inflation is at the Planck scale, $H \sim 1$. But then we encounter a different problem: there seems to be an upper bound on the amount of information that can be sent. Indeed, if the container is to survive a period of inflation at a high expansion rate H, then its diameter must be smaller than the horizon, $D < H^{-1}$, or else the container would be torn apart by the expansion. (One could imagine spreading the information among multiple containers, but it would be very difficult for it to be reassembled after the containers had been separated by vast distances during inflation.) Now, it has been argued that the amount of information contained in a sphere of diameter D is bounded by [13]

$$I \leq A/4 \tag{4}$$

where $A = \pi D^2$ is the surface area of the sphere (in natural units). The general validity of this "holographic" bound is still a matter of debate. However, for our particular case, the inequality (4) can be derived directly from the generalized second law of thermodynamics. The maximum information that a package can contain at a given value of its energy is equal to the logarithm of the number of microstates of this package compatible with the given energy. That is, the maximum amount of information coincides with the thermodynamic entropy of the package in thermal equilibrium. Now, we can think of a process in which the package collapses into a black hole (or is swallowed by a very small black hole). The entropy of the package has to be less than the entropy of the resulting black hole, which is equal to one fourth of its surface area [14]. Since the largest black hole that can exist in an inflating universe has radius $1/\sqrt{3}H$ [15], this implies that the largest amount of information that can be sent is

$$I_{\max} = 1/12H^2 \tag{5}$$

The usual values of H in inflationary models range from 10^{-7} for grand unification scale inflation to 10^{-34} for electroweak scale inflation, yielding $I_{\max} \sim 10^{13}$–10^{68}. This can be compared with $I \sim 10^{10}$ for the human genome, $I \sim 10^7$ for a typical book, and $I \sim 10^{15}$ for all books in the Library of Congress. Even if one makes no assumptions about the model, with $I \gtrsim 10^7$, Eqs. (5) and (3) require $H^2 \lesssim 10^{-8}$ and $\mathcal{P} \lesssim \exp(-10^8)$. This is an improvement over the case of nucleating bubbles, but still the number of attempts required to beat the odds far exceeds the number of elementary particles in the visible part of the universe (see footnote 5).

2.3. Negative Energies

The root of the problem appears to be in the tiny tunneling probability (3), so it is reasonable to inquire whether or not a new inflating region can be created without quantum tunneling. This question was addressed by Farhi and Guth [16], who concluded that the answer is "no," provided that a few very general assumptions are satisfied. Among these assumptions the most important is the weak energy condition, asserting that the energy density measured by any observer is never negative. Although it is satisfied in all familiar states of matter, this condition is known to be violated in certain states of quantum fields (e.g., the electromagnetic field or scalar fields that are used in inflationary models).

The newly created inflating region should have an extent $\gtrsim H^{-1}$, where H is the inflationary expansion rate. Hence, one needs to violate the weak energy condition in a spacetime region

$$\Delta L \sim \Delta t \gtrsim H^{-1} \tag{6}$$

The required magnitude of the negative energy density is

$$-\rho \gtrsim H^{-2} \tag{7}$$

For noninteracting fields, the magnitude and the duration of violations of the weak energy condition are constrained by the so-called quantum inequalities [17],

$$-\rho \lesssim (\Delta t)^{-4} \tag{8}$$

Combining this with Eqs. (6) and (7), we see that all conditions can be satisfied only for super–Planckian inflation with $H \gtrsim 1$. Then again, one has to face the information bound (5).

The negative energy density in Eq. (8) should be understood in the sense of a quantum expectation value. There are quantum fluctuations about this value, and occasionally the fluctuation may get large enough to provide the required negative energy in a sufficiently large region. But again, such fluctuations are suppressed by an exponentially large factor [18].

It should be noted, however, that the validity of quantum inequalities like (8) is not certain beyond the case of free quantum fields for which they have been established [19]. For example, the Casimir energy density [20] of the electromagnetic vacuum between two conducting plates appears to be negative and permanent, in violation of (8).

2.4. Limiting Curvature

Quantum effects at high curvature could significantly modify the dynamics of the gravitational field. In particular, it has been suggested [21] that there exists a limiting curvature $R_{\max}$ which can never be exceeded. This would result in a drastic change in the final stages of the gravitational collapse. It has been argued in ref. 21 that the black hole singularity and the adjacent high-curvature region in the black hole interior get replaced by a de Sitter space of the limiting curvature $R_{\max}$. In the Schwarzschild solution describing the usual black hole, the spacetime could not be continued beyond the singularity, but now the de Sitter space extends all the way into a new inflating universe which is in the absolute future with respect to the original one. Assuming that the state of limiting curvature is metastable and decays dumping its energy into particles, we will have formation of thermalized regions and the usual picture of eternal inflation inside the black hole.

In the course of the gravitational collapse, the effective energy-momentum tensor in this model should develop negative energy densities that violate quantum inequalities. In this sense, the limiting curvature conjecture can be regarded as a specific example of a more general class of models with negative energies. An important difference, however, is that with a limiting curvature, inflating universes automatically form inside black holes, with no effort required on our part. To send an information container to another universe, all we need to do is to drop it into a black hole. And in a recycling universe, black holes are no danger at all: they simply provide a passage to a new inflating region.

The curvature of de Sitter space is $R = 12H^2$, and it follows from (5) that the largest amount of information that can be sent is

$$I_{\max} = 1/R_{\max} \tag{9}$$

We thus see that a nontrivial amount of information can be sent only if $R_{\max}$ is well below the Planck scale, $R_{\max} \ll 1$.

2.5. Summary

To summarize, it appears that all mechanisms that involve quantum tunneling are doomed to failure because of extremely small tunneling probabilities. Creation of new inflating regions without quantum tunneling requires a violation of the weak energy condition that is in conflict with quantum inequalities. It is not clear how seriously this constraint is to be taken, since we do not know to what extent quantum inequalities apply to interacting fields. Since the future of the civilization depends on the outcome, this can be regarded as a good reason to increase funding for negative energy research! In the following section, we shall take an optimistic attitude and assume that advanced civilizations will figure out how to get around quantum inequalities.[7]

3. SURVIVAL STRATEGIES

Suppose now that we have resolved all "technical" problems associated with sending messages to future civilizations. This includes learning how to generate negative energy in a sufficiently large volume, so that we can create new inflating regions (in case such negative energies are not generated without our intervention, due to the limiting curvature or some other mechanism), and designing containers for information that can survive the negative energy density, a period of inflation, and the subsequent hot radiation era. Now, what should our strategy be? How many containers should we send? Should we search for a message in our part of the universe in case it was created by an advanced civilization that lived prior to our inflation?

Let us begin with the last question. The answer to that appears to be "no." Our thermalized region contains an infinite number of civilizations, but it can contain only a finite number of information packages from our predecessors. Hence, the probability for a package to be anywhere in our neighborhood is zero, and it would be foolish to waste our resources for a search.

[7] In the absence of energy conditions, wormholes [22] would also be possible, and perhaps could be used to communicate between different regions. However, the maintenance of a long-lasting wormhole requires negative energies to exist indefinitely, whereas creating a new inflating region as above requires them only for a short period of time. We will not consider wormhole scenarios any further here.

Having in mind that future civilizations are not going to look for our message, we could try to make the container very conspicuous. It could, for example, transmit its message in the form of electromagnetic waves, using some star as a source of energy. (Of course, the container should then be programmed to search for a suitable star.) Perhaps a more reliable plan might be for the "container" to instead be a device which reproduces our civilization in the new region, rather than waiting for new civilizations to evolve. In such a case one can consider this process to be the continuation of the old civilization, rather than a new civilization at all. One can even imagine some individual members of the old civilization surviving in the container into the new region, perhaps by having their physical form and state of knowledge encoded in some compact and durable way for later reproduction. However, note that the amount of information that can be included in the container is limited by the arguments of Section 2.2, so that it may be necessary to send "simplified" representatives if the energy scale of inflation is high.

A mission can be regarded a success if the information is successfully transmitted to or creates a civilization in the new region that is capable of sending messages of its own. If the probability of success is p, then one should send $N \gg 1/p$ information packages to make sure that at least one of them succeeds. The ultimate goal is to initiate an infinite tree of civilizations so that our knowledge or our civilization can propagate indefinitely into the future. If $\bar{N}$ is the average number of missions launched by the regions that form the tree, then the average number of regions in each successive generation is related to the number of their predecessors by a factor $p\bar{N}$. Thus, for $\bar{N} > 1/p$ there is a nonzero probability that the process will never end and the total number of civilized regions in the tree will be infinite.

ACKNOWLEDGMENTS

We are grateful to Eduardo Guendelman and Alan Guth for useful discussions. This work was supported in part by CIRIT grant 1998BEAI400244 (J.G.), by a NATO grant CRG 951301 (J.G.), and by the National Science Foundation (K.D.O. and A.V.).

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